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A capillary-tube grid model of an element of a porous medium is used to study the effect of microscale parameters of the medium on indices of the process of displacement of oil by a polymer solution.

The unidimensional displacement of oil by a polymer solution is described by the system of equations

$$m \frac{\partial s}{\partial t} + U \frac{\partial F}{\partial x} = 0,$$

$$m \frac{\partial}{\partial t} \left( s^* c + \frac{a}{m} \right) + U \frac{\partial}{\partial x} cF^* = 0,$$

$$F = f_1 \mu_2 / (f_1 \mu_2 + f_2 \mu_1), \quad F^* = f^* \mu_2 / (f^* \mu_2 + f_2 \mu_1).$$
(1)
(2)

A similar system was examined in [1]. The calculations explicitly considered the effect of an inacessible pore volume. The dependence of adsorption on saturation was ignored. To find the material functions F(s, c),  $F^*(s, c)$ ,  $s^*(s)$ , a(s, c) necessary for calculation, use was made of a model of a porous medium in the form of a bundle of parallel capillary tubes. A similarity solution was constructed which corresponded to the initial and boundary conditions:  $s(x, 0) = s_0$ ,  $c(x, 0) = c_0$ ,  $s(+0,t) = s^0$ ,  $c(+0,t) = c_{\perp}^0 = c^0/F^*(s^0, c_{\perp}^0)$ . It had the form  $s = s(\xi)$ ,  $c = c(\xi)$ ,  $\xi = mx/Ut$  and was determined from the following boundary-value problem:

$$\xi s_{\xi} = F_{\xi}, \tag{3}$$

$$\xi \frac{d}{d\xi} \left( cs^* + \frac{a}{m} \right) = \frac{d}{d\xi} (cF^*), \tag{4}$$

$$s(0) = s^0, \quad c(0) = c^0_+, \quad s(\infty) = s_0, \quad c(\infty) = c_0$$
 (5)

We will examine the same problem but will assume that the quantity of sorbed substance depends not only on the concentration c but on the saturation s. We change (4) and (5) to dimensionless form:

$$\xi \frac{d}{d\xi} (Cs^* + A) = \frac{d}{d\xi} (CF^*), \tag{6}$$

$$s(0) = s^{0}, \quad C(0) = 1, \quad s(\infty) = s_{0}, \quad C(\infty) = 0, A = a/[m(c_{+}^{0} - c_{0})], \quad C = c/(c_{+}^{0} - c_{0}).$$
(7)

In the region of continuity, the solutions of system (3), (6) are

$$\boldsymbol{\xi} = \{R_0 \pm [R_0^2 + 4(s^* + A_{,c})(F_{,s}F_{,c}C - F_{,s}(F^* + CF_{,c}))]^{1/2}\}/[2(s + A_{,c})],$$
(8)

$$R_0 = F_{,s}(s^* + A_{,c}) + F^* + CF_{,c}^* - F_{,c}\left(C\frac{ds^*}{ds} + A_{,s}\right)$$

We will study the effect of the relation A(s), i.e., we will set  $F^* = F$ ,  $s^* = s$ ,  $A_{,s} > 0$ . Then (see [2])

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Fig. 1. Relative phase permeabilities for: a) a hydrophilic porous medium; b) a partially hydrophobic (fraction of hydrophobic surface 0.5) porous medium; 1) oil; 2) water; 3, 4) water phase with R = 0.05 and R = 0.15, respectively; 5) polymer solution with R = 0.15.



Fig. 2. Dependence of the fraction of pore-space surface accessible for adsorption (solid lines) and polymer-solution saturation (dashed lines) on the saturation of the water phase for a hydrophilic (a) and partially hydrophobic (b) medium: 1) R = 0.05; 2) R = 0.15.

$$\xi = \{R_1 \pm [R_1^2 - 4 (s + A_{,c}) F_{,s} F_1^{1/2}] / [2 (s + A_{,c})], R_1 = F_{,s} (s + A_{,c}) + F - F_{,c} A_{,s}.$$
(9)

From (6), (9)

$$s'_{\xi}(\xi - F_{,s}) - C'_{\xi}F_{,c} = 0,$$
 (10)

$$C'_{s} = \frac{R_{2} \pm [R_{2}^{2} - 4(s + A_{.c})F_{.s}F_{.c}A_{.s}]^{1/2}}{2(s + A_{.c})F_{.c}},$$

$$R_{2} = F - F_{.s}(s + A_{.c}) - F_{.c}A_{.s}.$$
(11)

The same signs are taken in (9) and (11). A minus sign is chosen for nontriviality of C's in (11), since  $F_{,C} < 0$ ; here, from (9)  $\xi > 0$ . From (10)  $\xi = F_{,C}C'_{S} + F_{,S}$  with  $s_{\xi}^{i} \neq 0$ , from which we obtain the following after differentiation:

$$\xi'_{s} = F_{,CC} (C'_{s})^{2} + 2F_{,Cs} C'_{s} + F_{,C} C'_{s} + F_{,ss}.$$

If  $F_{,s} \rightarrow 0$ ,  $F_{,C} \rightarrow 0$  and  $F_{,ss} < 0$  as  $s \rightarrow s^{\circ}$ ,  $C \rightarrow 1$ , then there is a neighborhood of the point  $(s^{\circ}, 1)$  in which  $\xi_{s'} < 0$ , i.e., s and C continuously decrease in it with an increase in  $\xi$ .



Fig. 3. Distribution of saturation of the water phase (solid lines) and concentration of the polymer (dashed lines) in a similarity solution of a problem on the displacement of oil by water (1) and a polymer solution with R = 0.05 (2, 3) and R = 0.15 (4, 5) with allowance (3, 5) and without allowance (2, 4) for the relation A(s) and an inaccessible pore volume in a hydrophilic (a) and partially hydrophobic (b) bed.

This contrasts with the solutions obtained in [1-3], where no allowance was made for A(s) and the concentration of the polymer was decreased only abruptly.

In the general case the problem is solved numerically. As the material functions needed for the calculation it is possible to use the Langmuir sorption isotherm  $a(C, s) = \Gamma C / (1 + \Gamma C / a_{\infty}(s))$ ,  $a_{\infty}(s) = MSz_m\sigma(s)$  and the functions F(s, C),  $F^*(s, C)$ ,  $s^*(s)$ ,  $\sigma(s)$ , obtained by modeling an element of the porous medium with a stochastic grid of capillary tubes, a general diagram of which was presented in [4]. The capillary-tube-grid model reproduces such important geometric properties of the pore space of actual reservoirs as interconnectedness and variable pore size. Thus, it can be used to obtain a more adequate description of filtration processes than the model of parallel capillary tubes.

Besides the satisfaction of conditions of capillary equilibrium of the phases in the element, which is customary for problems of two-phase filtration, in the modeling we considered effects connected with the commensurateness of the dimensions of a macromolecular sphere (of radius R) and the pore channels. It was assumed that the polymer does not penetrate capillary tubes of radius less than R, while in pores filled with polymer solution macromolecular spheres undergo monolayer adsorption. The latter leads to a reduction in the hydrodynamic radius of the capillary tube by the amount  $\beta R$ , with a corresponding increase in hydraulic resistance. All of the linear dimensions were normalized on the mean radius of the capillary tube, i.e., it was taken as unity. The results presented below (Figs. 1 and 2) were obtained on a grid with the coordination number 6; the capillary-tube-radius distribution was taken to be a lognormal distribution with a mathematical expectation and dispersion equal to one. We used correlation of tube length with tube radius proposed by Fatt [4]:  $l = r^{-1}$ . The size of the grid was 30  $\times$  15. The viscosity of the polymer solution was taken as  $\mu^* = 3\mu_1$ . In the calculations we assumed  $\alpha = \beta = 2$ . As can be seen from Figs. 1 and 2, the following is true for these parameters: 1) use of the polymer significantly reduces the relative permeability of the water phase, the reduction being greater, the greater the value of R; 2) the effect of an unaccessible pore volume, causing s\* to differ from s and F\* to differ from F, is manifest significantly only on a hydrophilic medium with R = 0.15.

Figures 3 and 4 show results of a similarity solution for  $\mu_2/\mu_1 = 10$ ; m = 0.2;  $\mu^*/\mu_1 = 3$ ;  $\Gamma = 0.11$ ;  $MSz_m/c^0 = 1$ ;  $c_0 = 0$ . As  $f_1(s, 1)$  and  $f^*(s, 1)$  we used curves 3-5 in Fig. 1. The determination of F(s, C) and F\*(s, C) with  $C \in (0, 1)$  inevitably entails a certain degree of arbitrariness, so we adopted the simplest approximation: F(s, C) = (1 - C) F(s, 0) + CF(s, 1);  $F^*(s, C) = (1 - C) F(s, 0) + CF(s, 1)$ .

It is apparent from Figs. 3 and 4 that the following is true with the given model parameters: 1) in a hydrophilic porous medium, piston-like displacement of oil by water takes place, and use of the polymer does not change the saturation distribution in the similarity solution



Fig. 4. Dependence of oil recovery (solid lines) and the water content of the recovered oil (dashed lines) on the quantity of extracted liquid (in pore volumes) in a hydrophilic (1) and partially hydrophobic (2-6) medium from a sililarity solution of a problem on the displacement of oil by water (1, 2) and a polymer solution with R = 0.05(1, 3, 4) and R = 0.15 (1, 5, 6) with allowance (1, 4, 6) and without allowance (1, 3, 5) for the relation A(s) and an inaccessible pore volume.

(it should be remembered that we are dealing only with frontal displacement in a uniform medium); 2) in a medium made hydrophobic, use of the polymer significantly alters the saturation profile compared to displacement by water; a second saturation jump connected with the concentration jump appears; the running and final oil recovery both increase, the increase being greater, the larger R; 3) the effect of the relation  $\sigma(s)$ , leading to the appearance of a region of continuous reduction in concentration, predominates over the effect of an inaccessible pore volume; as a result, the concentration should increase; 4) a calculation which does not take into account the effect of the inaccessible pore volume and the relation  $\sigma(s)$ does not reproduce the region of continuous (although fairly slight) reduction in concentration and gives an understated value of the velocity of the polymer front; this error increases with an increase in R.

The above results indicate that the physical effects discussed here may significantly affect production indices and in some cases should be taken into account. It should be noted that with the chosen model parameters the relative phase permeabilities in the hydrophilic medium are shifted to the left compared to the hydrophobic medium. The shift of the relative phase permeability of the oil is particularly large, which leads to a reduction in oil recovery. With other parameters, such as with the use of the correlation  $l = r^{-2 \cdot 5}$ , the relative phase permeabilities are farther to the right for the hydrophilic medium than for the hydrophobic medium, and oil recovery is greater in the former.

## NOTATION

m, porosity; s, s\*, saturation of water phase and polymer solution; U, total filtration velocity of the water and oil phases; t, time; x, space coordinate; F, F\*, fraction of the water phase and polymer solution in the flow; c, C, dimensional and dimensionless concentrations of the polymer; a, A, dimensional and dimensionless quantity of polymer sorbed per unit volume of the porous medium; s<sub>0</sub>, c<sub>0</sub>, s<sup>o</sup>, c<sup>o</sup>, c<sup>o</sup><sub>+</sub>, constants determining the initial and boundary conditions;  $\xi$ , similarity variable; R<sub>0</sub>, R<sub>1</sub>, R<sub>2</sub>, notation used to abbreviate Eqs. (8), (9), and (11);  $\Gamma$ , constant;  $a_{\infty}$ , limiting adsorption of the polymer; M, mass of a polymer macromolecule; S, specific surface of the porous medium;  $\sigma$ , fraction of the surface of the porous medium; t dium available for adsorption;  $z_m$ , number of macromolecules adsorbed by a unit of surface in the dense monolayer; R, characteristic dimension of the macromolecules; l, r, length and radius of capillary tube;  $\alpha$ ,  $\beta$ , coefficients;  $\mu_1$ ,  $\mu_2$ ,  $\mu^*$ , viscosities of water, oil, and polymer solution; f<sub>1</sub>, f<sub>2</sub>, f\*, relative permeabilities of water, oil, and polymer solution.

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INFLUENCE OF AN EXTERNAL ELECTRIC FIELD ON THE PROPAGATION OF ULTRASOUND IN ELECTRORHEOLOGICAL SUSPENSIONS

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The influence of a constant external electric field on the velocity and attenuation of ultrasound in electrorheological suspensions is studied experimentally.

It is a well-known fact [1] that the elastic properties of liquids depend in different ways on the presence of polar or nonpolar solvents. An electric field, which orients the molecular dipoles, must affect the elastic properties of polar liquids. Such an influence can be detected by measuring the velocity of ultrasound [2]. Experimental and theoretical studies of the variation of the amplitude or phase of ultrasonic waves transmitted through a liquid in an electric field have been reported in a number of papers [1-6]. For example, Barone and Giacomini [3] have investigated the influence of an electric field on the orientation of constant molecular dipoles and also the variation of the viscosity and compressibility of "pure" polar liquids. Quartz oscillator crystals were immersed in nitrobenzene at a distance of 5 mm from a reflector. A slight variation of the oscillation frequency was observed when a potential of 4 kV was applied between the reflector and the forward electrode of the crystal. In this case the electric field vector was parallel to the direction of sound propagation. However, the observed frequency variation could not be unequivocally attributed to molecular orientation or heating of the medium. Nolle [4], proceeding from the premise of a variation of the sound velocity in liquids and the results of the experiments reported in [3], tested the hypothesis of phase modulation by an electric field in the case of perpendicular orientation. It was found that amplitude modulation is absent in all liquids, i.e., the acoustic attenuation is unaffected by an electric field. Phase modulation was observed in conducting liquids ( $v = 5 \cdot 10^6$  S/cm). It was inferred from the form of the phase modulation (quadratic dependence on E) that thermal effects play the dominant role. However, Nolle observed appreciable amplitude and phase modulation when an adsorbing material, glass wool, was placed in the liquid; this result was attributed to the mechanical motion of the glass fibers. The measurements were carried out within error limits of  $5 \cdot 10^{-4}$  dB/cm in a field of 1 kV/cm. Nolle's results were corroborated by Bonetti [5] in experiments with nitrobenzene, in which an electric field was not observed to have any influence on the velocity of ultrasound.

Thus, an analysis of the investigations reported to date shows that an electric field does not alter the amplitude and phase characteristics of sound propagating in "pure" liquids if the internal heating factor, which tends to diminish the field dependence of the ultrasound velocity in the majority of media, is eliminated. We have been unable to locate similar studies for disperse systems.

In our experiments we used electrorheological (ER) suspensions, for which it had been established previously in shear macroflows that the viscoplastic properties depend strongly

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